MATHEMATICAL MODEL FOR FAST COMPUTATION OF EROSION PROFILE IN SUBMERGED ARC FURNACE WITH FREEZE LINING

Hu Dong, Chu Shaojun

School of Metallurgical and Ecological Engineering, University of Science and Technology Beijing, Beijing, 100083, P.R.China, e-mails: 340166@qq.com, chushaojun@metall.ustb.edu.cn

ABSTRACT

Lining of submerged arc furnace is subject to chemical and physical erosion during smelting process. In this paper, a new 2D mathematical model is developed for monitoring the erosion profile of freeze lining in submerged arc furnace which is used for smelting ferronickel. Temperatures monitored by thermocouples at different positions of refractory are utilized for solving an inverse heat conduction problem, namely estimating the shape and location of the wear-line in melting furnace. In order to solve this inverse problem, the optimization procedure which is used to minimize the sum of the squared residual between calculated and measured temperature at different thermocouple locations should be executed with the aim to enhance computational efficiency. Temperatures at thermocouples' locations are calculated by Boundary Element Method. Modified simplex optimization method is employed. The result obtained from numerical experiments indicates that this algorithm could reduce the computational memory and time, at the same time improve the computational accuracy.

Furthermore, thermal conductivity of the refractory in freeze lining may decrease after smelting for a period of time. This is mainly due to refractories has experienced the thermal cycling for many times. The accuracy in computation of erosion profile will be affected either. The influence degree is also discussed in this paper.

KEYWORD: *Submerged arc furnace, freeze lining, erosion profile, boundary element method.*

1. INTRODUCTION

Submerged arc furnace (SAF) is still the main equipment for smelting Ferroalloys. The refractory of SAF is gradually destroyed by hot molten iron and slag due to physical and chemical erosion during the smelting process. Before the appearance of SAF with large capacity, not enough attention has been paid to the lifetime of SAF lining because the cost to relining is low. The investment to building linings of SAF with large capacity has been increased significantly and this project should be carefully prepared for several months.It is critical that continuous monitoring the residuary thickness of lining and estimating the precise time to blow down and relining.

In order to monitor the thickness of refractory accurately, some online monitoring methods have been developed to acquire the erosion profile of SAF since 2004. The method adopted in SAF is similar to the one that is used in Blast Furnace (BF). A large number of thermocouples are arranged in refractory to measure temperature and mathematical models that monitoring how wear-line is formulated. These models make use of measured temperature to seek the shape of erosion boundary. A.De Kievit [1] utilized the 1-d steady heat transfer method to monitor a SAF smelting ferromanganese. The date that blast furnace using 1-d method to monitor inner contour can trace back to 1966. Some corporations still employ this technique due to its simplicity and lower requirement for thermocouple displacement [2]. L.Rodd [3] developed a 2-d mathematical model to monitor a SAF smelting ferronickel. 1550°C isotherm is the wear-line in the temperature field, which is obtained by Finite Element Method (FEM). Karstein.S [4] exploited two types of numerical algorithms solving inverse heat conduction problem for an ilmenite melting furnace. One algorithm is based on utilizing a fixed boundary with control nodes. The other algorithmic approach is to approximate the wear-line as close as possible with as few curve representing parameters as possible.

Monitoring the wear-line of SAF is still at early stage compared with BF. Many effective monitoring methods have been developed, which mainly includes direct method [5-6] and numerical simulation [7-14]. In direct method, some sensors with the same thickness as refractory will be imbedded in the linings. The destructive progress of sensor is synchronized with lining erosion, so the residual thickness of sensors can reflect the erosion condition. This technique has the advantage of supplying extremely precise information but only at sensor location. And the implementation is difficult and costly. With respect to numerical simulation, the heat conduction equation describing mathematical model of lining is solved by some types of numerical method mainly including FEM, and the 1150°C isotherm in the temperature field is defined as erosion boundary. There are two aspects of heat transfer problem in numerical simulation: direct and inverse heat conduction problem. Boundary conditions and thermo-physical properties are known in direct heat conduction problem (DHCP) [7-9]. A remarkable limitation to the DHCP model need to be take noticed is that the domain area of model is not immutable. Hence it is necessary change the model along with the lining erosion. Either the thermo-physical properties or the boundary conditions can be known in inverse heat conduction problem (IHDP); instead, the temperature of the interior has to be known for some points in the domain [8]. Generally speaking, the shape of inner boundary in model is unknown. Some authors [10-12] parameterize the inner border and combine the optimization method with FEM to conquer the IHDP. However, because of the ill-posed property in inverse problem, regularization method is introduced to make the optimization process stable. Apart from FEM, Wu [13] attempted to employ the Boundary Element Method (BEM) to calculate the temperature field. Constructing 6 points to constitute the inner profile and adopting the orthogonal test to compute the positions of these points. Except for heat conduction model, computational fluid dynamics model was introduced to perform a numerical analysis of the BF hearth inner profile [14]. In brief, many excellent scholars have made a contribution to figuring out the complex erosion process of BF hearth. In addition to borrow ideas from monitoring method in BF, the particularity of SAF should also be considered carefully. Hearth of BF is always inbabited by molten iron. That region of SAF is occupied by molten metal-slag and charging alternately. Together with some other factors like that scheduling stoppages and intermediate repairs, the lining has inevitably experienced thermal cycling, which means the refractory temperature will rise and fall repeatly for many times. Furthermore, there are some differences about configurations between “Freeze lining” [15-16] (figure 1) of SAF and BF lining. The ceramic protection layer will be gone shortly after the furnace reaches full productivity [16]. With the carbon brick holding a high percentage in this system, the “Skull” will forms at the surface of that. Hence, the erosion degree of carbon brick can be known, it is possible to monitor the lining erosion of SAF.

In this paper, a new methodology that uses a 2d inverse heat conduction model and measured temperature obtained from thermocouples in the smelting process has been developed to estimate the erosion profile. The present model has a number of special characters by which differs from earlier ones. First, Kirchhoff transform is introduced to transfer the nonlinear differential equation to Laplace equation. In order to solve the differential equation faster, BEM and modified simplex optimization method is applied simultaneously. Second, the thermal conductivity of model is analyzed through the thermal cycling, whose influence on calculation accuracy of erosion boundary is also discussed. This methodology has been used to monitor an actual-scale submerged arc furnace smelting ferronickel with 16500 Kva at Shandong liangda factory.

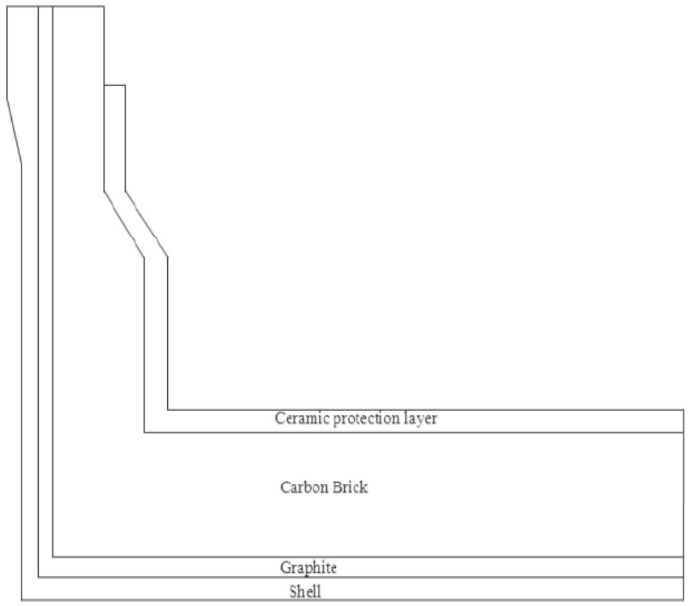


Figure 1: Configuration of submerged arc furnace Freeze lining

2. MATHEMATICAL MODEL OF DIRECT HEAT CONDUCTION

The computational domain Ω developed in this paper is illustrated in figure 2, which represents the region that occupied by carbon brick in the "freeze lining" system (figure 1). B_5 stands for the inner boundary of SAF, which is the target to calculate. B_2 and B_3 are interface between carbon brick and graphite at wall and bottom region, respectively. The heat source of SAF is comes from resistance heat and chemical reaction, which enters in Ω from B_5 and exports from B_2 and B_3.

So the heat flux through B_1 and B_4 is zero. In the interior of Ω region, 6 thermocouples are inserted to measure the temperature in the process (marked 1-6 in figure 2). Other 6 ones are located at the interface between carbon brick and graphite, the temperatures of which can be treated the boundary conditions of B_2 and B_3 after the interpolation. The steady state case under rotational symmetry is considered here:

$$\frac{\partial}{\partial z}(k(T)\frac{\partial T}{\partial z})+\frac{1}{r}\frac{\partial}{\partial r}(rk(T)\frac{\partial T}{\partial r})=0 \quad \text{in} \quad \Omega \tag{1}$$

where T represents the temperature at the point in domain Ω, r and z being the radial and axial coordinates,respectively. *K(T)* is the thermal conductivity of carbon brick, which generally depends on the temperature. This formula is always the nonlinear differential equation. It is difficult to deal with this equation by BEM in this case. So formula (1) should be transfered to Laplace equation by Kirchholf transform. The new parameter U is formulated to replace T.

$$U=\int_{T_0}^{T}\frac{k(T')}{k_0}dT' \tag{2}$$

$$\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial r^2}=0 \tag{3}$$

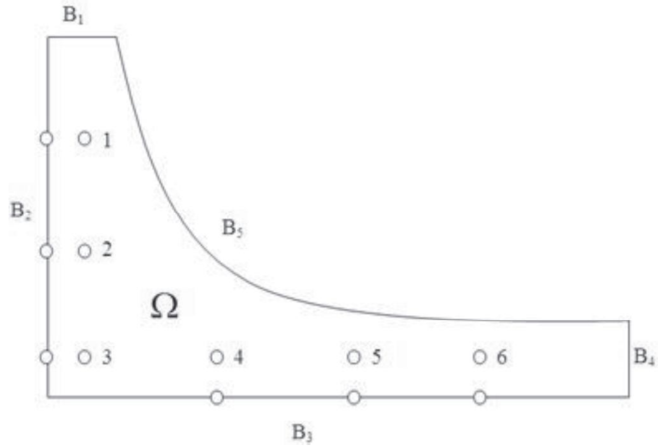


Figure 2: D physical model for computing erosion profile

K_0 is the thermal conductivity at temperature T_0. At the same time, boundary conditions should be revised. Dirichlet boundary conditions of B_2 and B_3 before transformation can refer to equation (4), where f is the function of temperature at B_2 and B_3.

$$T=f(r,z),\text{for}(r,z)\in(B_2\bigcup B_3) \tag{4}$$

$$U=\int_{T_0}^{f(r,z)}\frac{k(T)}{k_0}dT,\text{for }(r,z)\in(B_2\bigcup B_3) \tag{5}$$

At the boundary B_1 and B_4, insulation is assumed reasonable. The heat flux is still zero after transformation.

$$\frac{\partial U}{\partial r}=0,(r,z)\in B_1 \tag{6}$$

$$\frac{\partial U}{\partial z}=0,(r,z)\in B_4 \tag{7}$$

In the monitoring model for BF, 1150°C isotherm is the inner boundary. This temperature corresponds to the solidification of the eutectic Fe-C, and below this temperature, the hot metal is in the solid state [11]. On the basis of this usual principle and Fe-Ni binary phase diagram, the inner boundary in this model is 1550°C isotherm. The boundary conditions after transformation can refer to equation (9).

$$T=1550^{\circ}\text{C},\quad \text{for }(r,z)\in B_5 \tag{8}$$

$$U=\int_{T_0}^{1550}\frac{k(T)}{k_0}dT,\ \text{for }(r,z)\in B_5 \tag{9}$$

After Kirchholf transform, the Laplace equation describing domain Ω is convenient to solve by BEM. Before acquiring the temperature at the position of thermocouples, the unknown T and $\partial T/\partial N$ at boundary of domain Ω should be calculated.

$$\mathbf{AX} = \mathbf{F} \tag{10}$$

A and **F** matrix can be obtained by the discretization to boundary. **X** is the unknown matrix that includes the temperature of B_1 and B_4, the heat flow of B_2, B_3 and B_5. The temperature at the location of thermocouple can be calculated by equation (11).

$$T_C^i + \int_r Tq_{AS}^* dr = \int_r qT_{AS}^* dr \quad i = 1,2\cdots 6 \tag{11}$$

Where *Tc* is the calculated temperature in domain, integrating range r consist of whole boundary ($B_1 \cup B_2 \cup B_3 \cup B_4 \cup B_5$). The fundamental solution of Laplace equation is T_{AS}^*, which derivative is q_{AS}^*. In these equations, *d* is the distance between the Tc and domain boundary.

$$T_{AS}^* = \frac{1}{2\pi} \ln \frac{1}{d} \tag{12}$$

$$q_{AS}^* = \partial T_{AS}^* / \partial n \tag{13}$$

3. ALGORITHMS FOR INVERSE HEAT CONDUCTION

Prior to solving the concrete shape and location of inner surface, B_5 should be parameterized firstly. This processing scheme is described as follows (figure 3). Extending two straight lines of B_1 and B_4 boundary to intersect at a point A, and connecting n points that selected from B_2 and B_3 to point A, thus n "structure lines" (L_1,L_2..L_n) are formulated. B_5 will be formed by cubic spline interpolation from n "boundary points" (P_1,P_2..P_n), which are picked up at each structure line. One advantage of this formulation is that the rectangular region constituted by point A and origin O can includes any shape of B_5 boundary. Namely B_5 can be expressed by n boundary points coming from n structure line no matter what the shape of B_5 is. Because the figure of B_5 is determined by locations of boundary point, solving the inverse heat conduction problem is to seek the optimize location of boundary points.

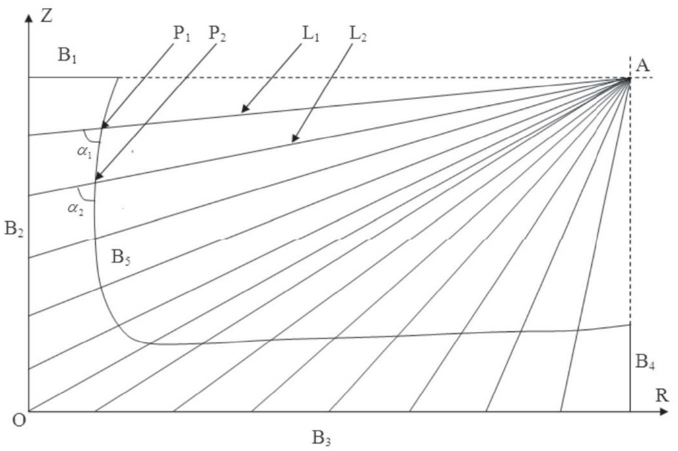


Figure 3: Construction scheme of inner boundary in computational model

In order to solve the inverse problem, the usual optimization method of minimizing the sum of the squared residual at 6 thermocouple locations is applied as many authors:

$$\min \Psi(p_1, \cdots p_n) = \min \sum_{i=1}^{6} (T_C^i - T_M^i)^2 \tag{14}$$

where T_M represents the measured temperature at thermocouple position. In the actual situation, the numbers of sensors is larger. The ill-posedness is a big obstacle to solve inverse problem. The small changes in the measured data correspond to big changes in the solution of equation (14). There is inherent error in the measuring process of thermocouples in addition. So in order to prevent the emergence of unreliable solution, the regularization method is adopted to amend the equation (14). Therefore, the equation is minimized with a regulizer as literature [10-11].

$$\min \Psi(p_1, \cdots p_n) = \min(\sum_{i=1}^{6} (T_C^i - T_M^i)^2 + \gamma \sum_{j=1}^{n} (\alpha_{j+1} - \alpha_j)^2) \quad j = 1, \cdots n \tag{15}$$

where j is the number of structure lines in the model, α_j is the angle that structure line and B_5(see figure 3), and γ is a regularization term. The solving method to equation (15) comprises of first or second derivative to optimize parameters and direct searching method. The process that derivative to parameter ($P_1,P_2..P_n$) is the derivative of the composite function to multi-element equations set. This procedure is complicated and a great deal of calculation is needed. The modified simplex method [17] belonging to direct searching procedure overcomes such fault and has the rapidly converging feature. A normal simplex should be built at first (equation 16), one peak point (p^i) of which is a group positions of boundary points (equation 17).

$$P = [p^1, p^2 ... p^{n+1}]^T \tag{16}$$

$$P^i = [p_1, \cdots p_n], \ \ \mathrm{i} = 1,2...\mathrm{n}+1 \tag{17}$$

At each round of simplex iterative computation to each peak point, the maximum value and minimum value are indicated by Ψ_{MAX} and Ψ_{MIN}. The corresponding peak point are marked by P^{MAX} and P^{MIN}. Furthermore, the center point of mass x_C, reflection point x_R, compression point x_S and expansion point x_E should be calculated as follows.

$$x_C = \frac{1}{9}(\sum_{i=1}^{n+1} p^i - p^{MAX}) \tag{18}$$

$$x_R = 2x_C - x^{MAX} \tag{19}$$

$$x_S = 0.5(x^{MAX} + x_C) \tag{20}$$

$$x_E = 2x_R - x_C \tag{21}$$

The iteration stopping criterion of optimization process is showed at equation (22), where ε is convergence criteria. Finally, the computation flow that combines BEM and modified simplex method is exhibited in figure 4.

$$\{\frac{1}{n+1}\sum_{i=1}^{n+1}[\Psi(p^i)-\Psi(x_C)]^2\}^{1/2} \le \varepsilon \tag{22}$$

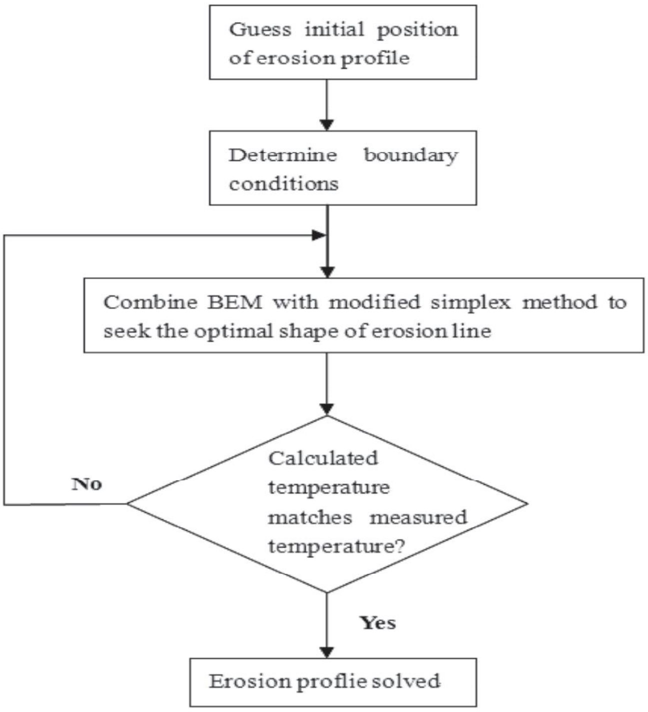


Figure 4: Flow chart of computational model

4. THE INFLUENCE OF THERMAL CYCLING TO THERMAL CONDUCTIVITY

The carbon brick in SAF lining will inevitably suffer from through thermal cycling for many times. At once routine blow down of SAF smelting ferronickel with 16500KVA at Liangda, two carbon bricks that newly manufactured and used for a period of time are measured by NETZSCH LFA 457 Micro laser Flash apparatus. The detection result is showed in figure 5. The thermal conductivity of carbon brick descends average 8 W/(m·°C) compared to the one before thermal circulation. However, the function relation between thermal conductivity and temperature is unchanged, is still similar to a exponential function. This variation may affect the computational result to erosion profile, which are described in the following subsections.

5. RESULTS AND DISCUSSION

Considering engineering safety, the highest measured temperature of 6 thermocouples within one month is selected to calculate a “severe wear-line”. Like differential equation and boundary conditions, the data of 6 thermocouples in domain is also needed to be transferred by Kirchhoff method. The impact of thermal cycling on thermal conductivity is also considered. Table 1 illustrates the result that transferred from these two situations.

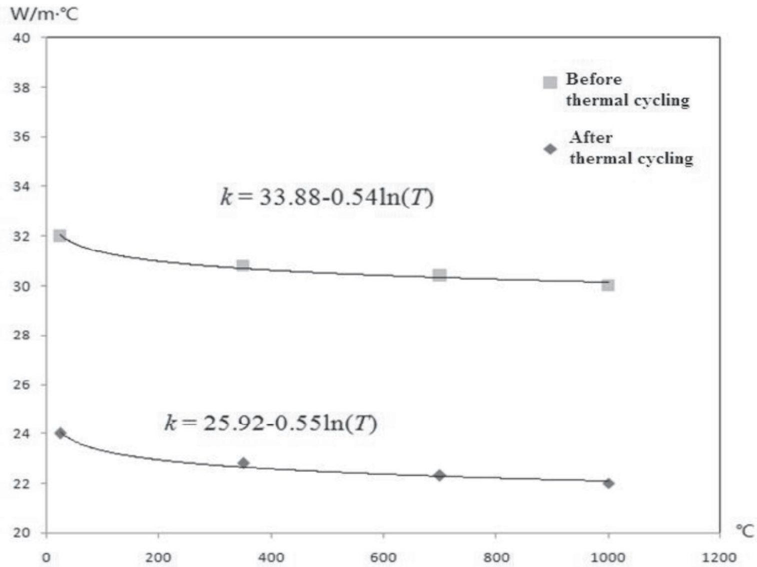


Figure 5: Variety of thermal conductivity under thermal circulation

Table 1: Kirchhoff transfer temperature at thermocouple location

		Point 1	Point 2	Point 3	Point 4	Point 5	Point 6
Temperature(℃)	measured	733.58	677.39	333.46	638.87	755.11	773.29
Before thermal cycling	transferred	676.83	623.56	295.44	586.99	697.23	714.44
After thermal cycling	transferred	667.41	615.17	292.64	579.30	687.40	704.27

The methodology in this paper is programed in MATLAB (R2010a). The convergence criteria ε is chosen as 25 and regulizer term γ is defined as 0.015. This program will converge in 2 minutes as the result of discretization only to boundary in BEM and modified simplex method without derivation. Table 2 gives the comparison of measured and calculated temperature. It can be seen that the temperature difference between two cases is less than 5°C.

Table 2: Measured and computational temperature at thermocouple location

Temperature, ℃		**Point 1**	**Point 2**	**Point 3**	**Point 4**	**Point 5**	**Point 6**
Before thermal cycling	Measured	676.83	623.56	295.44	586.99	697.23	714.44
	Calculated	677.39	625.90	295.46	587.08	696.27	715.74
After thermal cycling	Measured	667.41	615.17	292.64	579.30	687.40	704.27
	Calculated	669.65	618.09	287.87	578.17	687.76	705.61

The figure 6 shows the result of calculated wear-line in both cases. The erosion degree of the monitoring model that has experienced thermal circulation is more severe. At the corner and bottom region of hearth, the inner boundary distance of these two models is about 11cm that account for 7% of bottom thickness. The obtained results point out a main issue: the thermal conductivity will decline after the thermal cycling, but the "Skull" need to be emerged at the inner surface with the purpose of protecting lining from erosion; a certain heat is required to export from hearth; In terms of 1-d stable heat conduction equation (see equation (23), the thickness of refractory (δ) should be decreased because of the decline of thermal conductivity (λ). So the model developed to monitoring the erosion profile of SAF should be corrected after a period of smelting time. The temperature field of these two models is illustrated at figure 7.

$$Q = \frac{\Delta T}{\delta/\lambda} \tag{23}$$

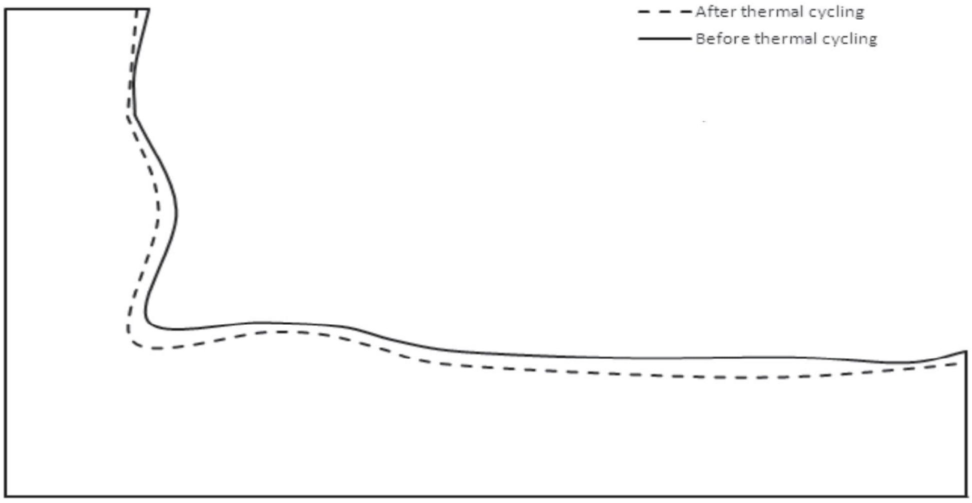
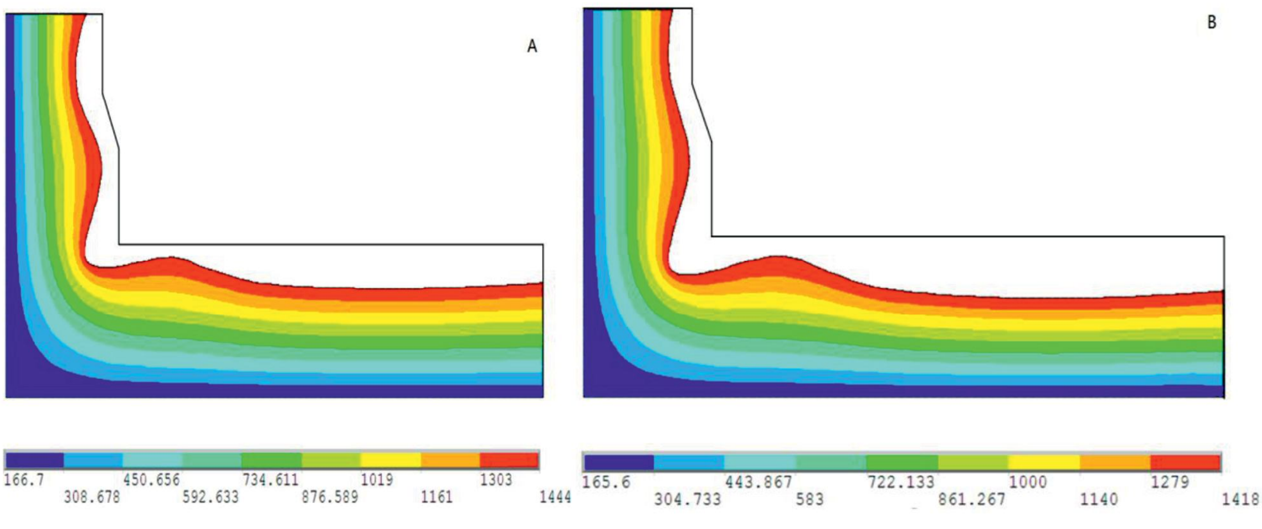


Figure 6: Computing result of lining erosion profile



(A)Before the thermal cycling (B)After the thermal cycling

Figure 7: The temperature filed of monitoring model

6. CONCLUSIONS

A new inverse heat conduction computational model combining BEM with modified simplex method is developed to monitor the erosion profile of linings, which is implemented to a SAF smelting ferronickel with the capacity of 16500KVA at Liangda factory. After the wear-line obtained by this methodology, accuracy of results is validated through by comparing calculated temperature with measured temperature. The difference in temperature can be restricted within 5°C.

The discrepancy between SAF and blast furnace in monitoring erosion is also discussed in this paper. In the "freeze lining" of SAF, carbon brick accounts for a high proportion of lining configuration and the "Skull" is formed at its surface. So the monitoring model only including carbon brick is developed. The carbon brick in lining system endures thermal cycling for many times during smelting in SAF. This procedure makes the thermal conductivity of carbon brick decrease about 8 W/(m·°C). The computing result of lining erosion profile is affected too. At the bottom and corner of hearth, the lining thickness difference of these two models with different thermal conductivity is about 11cm accounting for 7% of bottom thickness. So the model developed to monitor the erosion profile needs to be modified after a period of smelting time. The monitored erosion in SAF is at starting stage. Many problems remain to be solved. In the future, the lining monitoring model with the property of SAF should be developed.

7. REFERENCES

[1] A. De Kievit, S. Ganguly, P. Dennis. Monitoring and control of furnace 1 Freeze lining at Tasmanian electro-metallurgical company[C]. The 10th international ferroalloys congress proceedings.Cape Town： INFACON, 2004.pp. 477-483.

[2] Kuncan Zheng, Zhi Wen, Xunliang Liu. Research status and development trend of numerical simulation on blast furnace Lining erosion[J]. ISIJ International.2009.49(9):1277-1282.

[3] L.Rodd, N.Voermann, F.Stober. SNNC: a new Ferro-Nickel smelter in korea[C]. The 12th international Ferroalloys Congress Proceedings. Helsinki： INFACON, 2010.pp:698-700.

[4] Karstein.S,Inge M.Skaar.Monitoring the wear-line of a melting furnace[C].3rd Int.Conference on inverse problems in engineering. Port Ludlow,USA.1999:1-11.

[5] Gu Jie,Zhu He.Analysis for the vertical single-pipe heating system feature[J].Journal of Baotou university of iron and steel technology.1997.16(9): 241-246.

[6] Gao Zhengkai, Huang Luan.Measurement of lining thickness in blast furnace by electric resistance method and its application in No.1 BF of Shanghai Baoshan Iron and steel complex[J].Iron and Steel.1992.27(3):6-9.

[7] H.B Zhao, S.S Cheng and M.G Zhao.Analysis of all-carbon brick bottom and ceramic cup synthetic hearth bottom[J].Journal of iron and steel research International.2007.14(2):6-12.

[8] M.Swartling,B.Sundelin,A.Tilliander,P.G.Jonsson. Heat Transfer Modelling of a Blast Furnace Hearth[J].Steel Research.2010.3:186-196.

[9] Surendra KUMAR. Heat Transfer Analysis and Estimation of Refractory Wear in an iron Blast Furnace Hearth Using Finite Element Method[J].ISIJ International.2005.45(8):1123-1128.

[10] Johnny Brännbacka, Henrik Saxén.Model for Fast Computation of blast furnace hearth erosion and buildup profiles[J].Int.Eng.Chem.Res.2008.47.7793-7801.

[11] M.Zagaria,V.Dimastromatteo,V.colla.Monitoring erosion and skull profile in blast furnace hearth[J].Ironmaking and Steelmaking.2010.37:229-234.

[12] Jan TORRKULLA, Henrik Saxén.Model of the state of the blast furnace hearth[J]. ISIJ International.2000.40(5):438-447.

[13] Lijun Wu,Huier Cheng.Mathematical model for on-line prediction of bottom and hearth of blast furnace by particular solution boundary element method[J].Applied Thermal Engineering.2003.(23):2079-2087.

[14] Yu Zhang,Robit Deshpande.Numerical analysis of blast furnace hearth inner profile by using CFD and heat transfer model for different time periods[J].International Journal of Heat and Mass Transfer.2008.51:186-197.

[15] Athony M.Heam,Albert J.Dzermejko,Pieter H.Lamout.“Freeze”lining concepts for improving submerged Arc Furnace Lining Life and Performance[C].The 8th international Ferroalloys Congress Proceedings.BeiJing:INFACON,1998.pp:401-402.
[16] P. L. Duncanson, J. D. Toth. The truths and myths of freeze lining technology for submerged arc furnace[C]. The 10th international ferroalloys congress proceedings. Cape Town：INFACON, 2004.pp:488-499.
[17] Himmelblau.Applied nonlinear programming [M], Mcgraw-Hill Publishers, 1972.